Announcements: • Midterm

When: Monday, Feb 11, 7:30 - 8:30 PM

Where: BUCH A101

No notes / calculators.

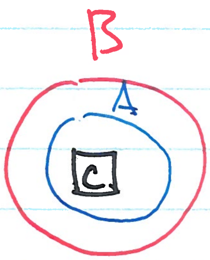
The best way to practice is past midterms, review HW from the last few weeks.

Google search "Rachel Ollivier Math 220"

- HW #5: deadline midnight tonight ⟵ same place
- HW #6: will only be due after break (with HW #7)

(Reminder: HW can be picked up in the Math Learning Center, LSK 300)

Exercise 2: ($\Longrightarrow$) Suppose $A \subseteq B$.



Then for every set $C \subseteq A$, we have

$C \subseteq A$ and $A \subseteq B \implies C \subseteq B$ (proved Monday).

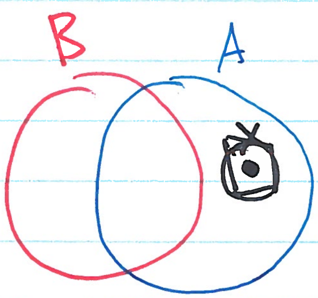
So we've shown

if $C \in P(A)$ then $C \in P(B)$.

So $P(A) \subseteq P(B)$.

($\Longleftarrow$) I will prove the contrapositive, i.e.

if $A \not\subseteq B$ then $P(A) \not\subseteq P(B)$



Because A is not a subset of B, there's some $x \in A$ such that $x \notin B$.

Then $\{x\}$ is a subset of A but not a subset of B.

Then $\{x\} \in P(A)$, $\{x\} \notin P(B)$.

Therefore $P(A) \not\subseteq P(B)$. ∎

Another proof of ($\Leftarrow$): If $P(A) \subseteq P(B)$, then because $A \in P(A)$, we have $A \in P(B)$. Thus $A \subseteq B$. ∎

Exercise 1: Let A, B be sets. Prove

$$A \times B \text{ is empty} \iff A \text{ is empty or } B \text{ is empty}$$

Exercise 2: Let A, B be sets. Prove

$$A \subseteq B \iff P(A) \subseteq P(B)$$

Exercise 3: Prove

$$A \times (B \cup C) = (A \times B) \cup (A \times C).$$

Exercise 4: Which is bigger? $P(A \times B)$, or $P(A) \times P(B)$?

Exercise 3: We will show that for every (x,y),

$$(x,y) \in A \times (B \cup C) \iff (x,y) \in (A \times B) \cup (A \times C)$$

def of ~~(×)~~ cartesian product

Proof: $(x,y) \in A \times (B \cup C) \equiv (x \in A) \text{ and } (y \in B \cup C)$

def of union

$$\equiv (x \in A) \text{ and } (y \in B \text{ or } y \in C)$$

distributive

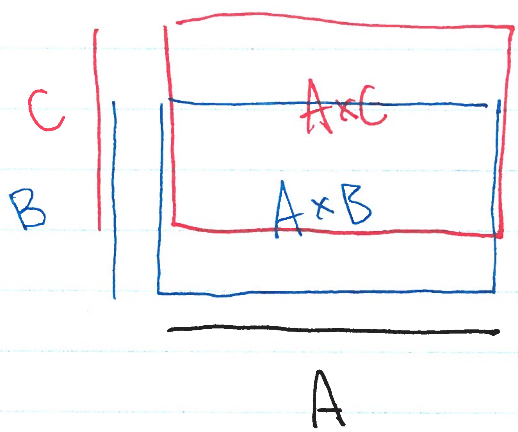
$$\equiv (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C)$$

def of Cartesian product

$$\equiv ((x,y) \in A \times B) \text{ or } ((x,y) \in A \times C)$$

def of union

$$\equiv (x,y) \in (A \times B) \cup (A \times C) \quad . \blacksquare$$



Q: Which is bigger: $P(A\times B)$ or $P(A)\times P(B)$

In general: $P(A\times B)$ is bigger $P(A)\times P(B)$

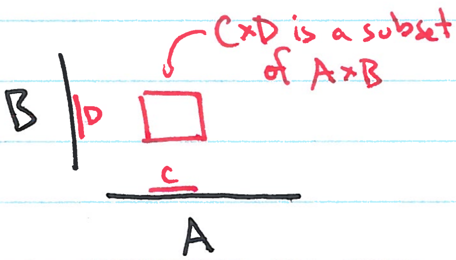
much bigger

Ex: $A=\{1,2,3\}$ $\quad B=\{4,5,6\}$

then $P(A)\times P(B)$ has $8\times 8 = 64$ elements

$P(A\times B)$ has $2^9 = 512$ elements

Note that $P(A)\times P(B)$ has some elements like $\phi\times\{4\}$, $\phi\times\{5\}$, $\phi\times\{6\}$, etc. which all "look like" the same element $\phi\in P(A\times B)$, but are distinct. Thus, $P(A\times B)$ doesn't actually contain $P(A)\times P(B)$



If A has size $m\in\mathbb{N}$ and B has size $n\in\mathbb{N}$, then

$P(A\times B)$ has size 2^{mn}

$P(A)\times P(B)$ has size $2^m\cdot 2^n = 2^{m+n}$

2^{mn} is generally bigger than 2^{m+n}.